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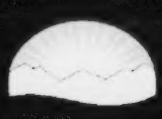
Amabilis Fir Height–Age and Growth Intercept Models for British Columbia

Gord Nigh

2009



Ministry of Forests and Range
Forest Science Program

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BRITISH COLUMBIA
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ABSTRACT

The purpose of this project was to develop growth intercept and height–age models for amabilis fir (*Abies amabilis* Dougl. ex Forbes) on the coast of British Columbia. Twenty-eight stem analysis plots were established on Vancouver Island. These data were combined with data from 46 previously established stem analysis plots. The stem analysis data were converted into height-breast height age data and the site index was obtained from the data. A suite of 50 growth intercept models was developed, one model for each breast height age from one to 50. A height–age model based on the Hossfeld IV function was also developed. There were minor differences between the new height–age model and the height–age model currently in use. Therefore, implementation of the new height–age models should have minimal consequences on timber supply analyses and forest management decision making.

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INTRODUCTION

Amabilis fir (*Abies amabilis* Dougl. ex Forbes), also known as Pacific silver fir or silver fir (Burns and Honkala 1990), is a common species along the coast of British Columbia (B.C.). It is found mainly in the Coastal Western Hemlock (CWH) and Mountain Hemlock biogeoclimatic zones, from the southernmost parts of the CWH to as far north as Stewart, B.C., and to a lesser degree in the northern wetter submaritime Interior Cedar Hemlock zone (Meidinger and Pojar 1991; Klinka et al. 2000). Amabilis fir is very shade-tolerant, and consequently can be found in all seral stages of a stand and with one or more other species (Klinka et al. 2000). Because of its large moisture requirements, it grows mainly on sites that receive high levels of precipitation (Klinka et al. 2000).

Amabilis fir is a relatively important timber species on the coast of British Columbia. In 2006/07, just under two million m^3 of balsam fir[1] was harvested, representing approximately 9% of the total volume harvested in the Coast Forest Region (British Columbia Ministry of Forests and Range and Minister Responsible for Housing 2007). Relatively little amabilis fir is planted, but it regenerates on a variety of seedbeds and is very shade-tolerant so natural regeneration is a viable option (Klinka et al. 2000).

1 Balsam fir includes all fir species; however, amabilis fir is by far the predominant fir species on the coast.

Very little information is available on site index-related growth and yield for amabilis fir on the coast of British Columbia. Kurucz (1982) published a report on the development of site index curves for amabilis fir. The methods used in his research are somewhat dated; the current data collections standards are different. As well, it is not clear where the sample plots were located. The growth intercept method (e.g., Wakeley and Marrero 1958; Alban 1972; Nigh 1997) had been developed by the time Kurucz did his work, but its use was not a standard practice in British Columbia. As a result, growth intercept models were not developed for young stands. Therefore, the development of new height-age and growth intercept models is warranted.

The purpose of this project was to develop growth intercept models and a new height–age model for amabilis fir, and to supplement the Site Index–Biogeoclimatic Ecosystem Classification (SIBEC) data warehouse with new amabilis fir

data.[2] The intent of the data collection was to cover the range of coastal amabilis fir using new data collection standards. The reanalysis of the height-age model provided the opportunity to employ the latest techniques for analyzing height–age models.

2 For more information about SIBEC, go to www.for.gov.bc.ca/hre/sibec/.

DATA

The targeted number of sample plots was not achieved due to budget constraints and the difficulty in finding unsuppressed trees. Therefore, the data set collected for this project was supplemented with that of Kurucz (1982).

The Kurucz (1982) data set is described briefly here to contrast it with the new data set that was collected specifically for this project. The Kurucz data set consisted of 50 plots (47 extant) with four sample trees in each plot. The sample trees were selected from the 20% largest diameter trees in a 0.1 ha plot. A section of each sample tree was taken at breast height (1.3 m) and at 10 equally spaced positions along the stem from breast height to the top of the tree. The rings in each section were counted. The height-ring count data were converted into height–breast height age data while making a correction for stem analysis bias (Carmean 1972; Newberry 1991). The data were then linearly interpolated between sectioning points to get annual height–breast height age pairs, which were then averaged by plot to get annual plot height–breast height age data. The plot height at breast height age 50 is, by definition, the site index.

The new data set consisted of 41 sample plots located on Vancouver Island. The SIBEC data collection standards (see www.for.gov.bc.ca/hre/sibec/documents/standards.pdf) were followed when establishing plots. Provided it met all the criteria for a site tree (see the SIBEC data collections standards for details about the criteria), the largest diameter tree from the 0.01 ha site index plot was felled and split open longitudinally to reveal the pith nodes. The heights of the pith nodes above breast height were measured, resulting in a height–breast height age data set for each tree. A full ecosystem classification (Luttmerding et al. 1990) was done on each plot using the Ground Inspection Form (form FS212-2).

METHODS

The two data sets were combined and the height trajectories for each sample plot were graphed. At this point, the data consisted of a series of height and breast height age pairs for each plot from breast height age one up to the greatest breast height age that was recorded. The height trajectories were examined for erratic or slow growth followed by a rapid release in height growth, which is evidence of suppression, stem breakage, or other problems that would indicate that the sample trees were not site trees. Fourteen such plots were identified and deleted from the data set, 13 of which were from the new data set. Finding unsuppressed trees was a problem in both studies. This left 74 plots of data for developing the growth intercept and height–age models. Kurucz did not do an ecosystem classification on his plots, so they could not be used to supplement the SIBEC data warehouse, nor could tests be done to detect differences in growth patterns on different ecosystems. Table 1 contains summary information about the sample plots.

TABLE 1 *Average breast height age, height, and site index for the study plots by subzone variant and site series*

Zone	Subzone variant	Site series	No. of plots	Breast height age (yrs)	Height (m)	Site index (m)
Unknown	Unknown	Unknown	48	106	37.69	25.67
CWH	vm1	01	11	86	38.13	28.23
CWH	vm1	03	1	87	31.88	22.58
CWH	vm1	05	2	80	38.82	29.78
CWH	vm1	06	4	78	38.06	29.53
CWH	vm1	07	1	70	37.60	33.21
CWH	vm2	01	4	81	32.39	24.08
CWH	vm2	03	2	86	30.58	21.49
CWH	vm2	07	1	68	34.05	26.56

GROWTH INTERCEPT MODEL DEVELOPMENT

The development of the growth intercept models followed that of Nigh (1997). The growth intercept is the average annual height growth of the site tree(s) above breast height. The growth intercept was calculated for each age from breast height age one to 50. The formula for the growth intercept is given by Equation 1:

$$GI_A = \frac{H_A - 1.3}{A - A_1} \times 100 \qquad (1)$$

where GI_A = growth intercept (cm/yr) for $A = 1, 2, \ldots, 50$ (yrs),
A = breast height age (yrs),
H_A = average height (m) of the site tree(s) at breast height age A,
A_1 = proportion of height growth between breast height ages zero and one that occurred below breast height = $(1.3 - H_0)/(H_1 - H_0)$,
H_0 = average height (m) of the site tree(s) at the end of the growing season immediately before the tree(s) reached breast height, and
H_1 = average height (m) of the site tree(s) at breast height age one.

If H_0 was unknown, then A_1 was assumed to be 0.5. The growth intercept model is:

$$SI_i = 1.3 + b_0 \times GI_{A,i}^{b_1} + \varepsilon_i \qquad (2)$$

where SI_i = site index (m) for the site tree(s) in plot i,
$GI_{A,i}$ = growth intercept (cm/yr) of the site tree(s) in plot i,
b_0, b_1 = model parameters, and
ε_i = random error term for plot i.

This model was separately fit to each breast height age from one to 50. Parameters were estimated using maximum likelihood with the procedure NLMIXED in SAS (SAS Institute Inc. 2004). The standard assumptions were tested with the Kolmogorov-Smirnov D statistic to test for normality of the

residuals (Stephens 1986), a t-test to verify that the residuals had a mean of zero, and plots of the residuals versus the predicted site index and the growth intercept to check for homoscedasticity.

HEIGHT-AGE MODEL DEVELOPMENT

The development of the height-age model generally followed that described in Cieszewski (2003). The full data set was reduced by deleting data for breast height ages that were not a multiple of 5. It is not necessary to analyze all of the data, and deleting data in this way reduces serial correlation (Davidian and Giltinan 1995, p. 315). The model that was fit to the data was based on the Hossfeld IV function (Cieszewski 2003):

$$H_{i,j} = 1.3 + (SI_i + u_i - 1.3) \times \left(\frac{(b_0 + R_i \times 49.5^{b_1 - 1}) \times (BHA_{i,j} - 0.5)^{b_1}}{(b_0 + R_i \times (BHA_{i,j} - 0.5)^{b_1 - 1}) \times 49.5^{b_1}} \right)^{1/3} + \varepsilon_{i,j} \quad (3)$$

where $R_i = \varphi_i + \sqrt{\varphi_i^2 + 2 \times b_0 \times (SI_i + u_i - 1.3)^3 / 49.5^{b_1}}$,

$\varphi_i = (SI_i + u_i - 1.3)^3 / 49.5 + b_2$,

$H_{i,j}$ = average height (m) of the site tree(s) in plot i at measurement j,

SI_i = site index (m) of sample plot i,

u_i = a unique (local) parameter for plot i which is modelled using indicator variables,

b_0, b_1, and b_2 = model parameters that are the same across all plots (global parameters),

$BHA_{i,j}$ = breast height age (yrs) of the site tree(s) in plot i at measurement j,

$\varepsilon_{i,j}$ = error term = $\rho_1 \times \omega_{i,j-1} + \rho_2 \times \omega_{i,j-2} + \omega_{i,j}$,

ρ_i = correlation parameter, and

$\omega_{i,j}$ = an identically and independently distributed $N(0, \sigma^2)$ random error term for plot i at measurement j.

The terms $\rho_1 \times \omega_{i,j-1}$ and $\rho_2 \times \omega_{i,j-2}$ model the first- and second-order autocorrelation (Sen and Srivastava 1990; Cieszewski 2003) using the previous two measurements (i.e., 5 and 10 years previously). $\omega_{i,j-1}$ and $\omega_{i,j-2}$ are the residuals from the previous two observations. If $\omega_{i,j-1}$ or $\omega_{i,j-2}$ was not available, then it was set to zero. Model 3 requires a base age 50 site

index; hence, the term 49.5 in the model. This is the number of growing seasons the tree has experienced after it reached breast height because the first growing season is only ½ year on average.

Model 3 was fit using maximum likelihood in procedure NLMIXED in SAS (SAS Institute Inc. 2004). The residuals ($\omega_{i,j}$) were used to test the following standard assumptions: a quantile-quantile plot tested for normality; a t-test checked for a mean of zero; and plots of the residuals versus the predicted height, breast height age, and site index tested confirmed a constant variance. The Durbin-Watson test for serial correlation was also applied to the residuals to determine whether the autocorrelation was successfully modelled.

RESULTS

A suite of growth intercept models for breast height ages one to 50, and a height-age model resulted from these analyses.

GROWTH INTERCEPT MODEL RESULTS

The fitted growth intercept models are presented in Table 2. This table also presents the square root of the mean squared error (RMSE) for each model. The RMSE gives an indication of how much error in the site index estimate can be expected when using the growth intercept model. For all ages, the mean of the residuals from the growth intercept model were not significantly different from 0 at $\alpha = 0.05$, and the plots of the residuals versus the growth intercept and the predicted site index did not show any signs of heteroscedasticity. However, the models for breast height ages 5, 7, 47, 49, and 50 showed slight evidence of non-normality in the residuals.

HEIGHT–AGE MODEL RESULTS

The parameter estimates and their standard errors that resulted from the fitting of the height-age model are shown in Table 3. The estimate for parameter b_2 was not significantly different from 0 at $\alpha = 0.05$, and therefore was deleted from the final

TABLE 2 *Fitted growth intercept models and the square root of their mean squared error (RMSE) for breast height ages one to 50*

Breast height age (yrs)	Model	RMSE	Breast height age (yrs)	Model	RMSE
1	$SI=1.3+12.14\times GI^{0.1957}$	4.250	26	$SI=1.3+1.171\times GI^{0.7629}$	1.450
2	$SI=1.3+10.29\times GI^{0.2324}$	4.139	27	$SI=1.3+1.113\times GI^{0.7758}$	1.383
3	$SI=1.3+8.348\times GI^{0.2829}$	3.952	28	$SI=1.3+1.039\times GI^{0.7933}$	1.308
4	$SI=1.3+6.151\times GI^{0.3585}$	3.624	29	$SI=1.3+0.9762\times GI^{0.8092}$	1.253
5	$SI=1.3+5.243\times GI^{0.3967}$	3.454	30	$SI=1.3+0.9295\times GI^{0.8217}$	1.213
6	$SI=1.3+4.009\times GI^{0.4616}$	3.166	31	$SI=1.3+0.8831\times GI^{0.8349}$	1.173
7	$SI=1.3+3.561\times GI^{0.4893}$	2.983	32	$SI=1.3+0.8522\times GI^{0.8444}$	1.133
8	$SI=1.3+3.313\times GI^{0.5064}$	2.895	33	$SI=1.3+0.8202\times GI^{0.8548}$	1.062
9	$SI=1.3+3.010\times GI^{0.5293}$	2.729	34	$SI=1.3+0.7936\times GI^{0.8639}$	1.010
10	$SI=1.3+2.667\times GI^{0.5588}$	2.565	35	$SI=1.3+0.7720\times GI^{0.8716}$	0.9480
11	$SI=1.3+2.436\times GI^{0.5810}$	2.462	36	$SI=1.3+0.7304\times GI^{0.8864}$	0.8841
12	$SI=1.3+2.265\times GI^{0.5988}$	2.362	37	$SI=1.3+0.6982\times GI^{0.8986}$	0.8243
13	$SI=1.3+2.120\times GI^{0.6152}$	2.297	38	$SI=1.3+0.6690\times GI^{0.9101}$	0.7810
14	$SI=1.3+1.999\times GI^{0.6301}$	2.236	39	$SI=1.3+0.6367\times GI^{0.9236}$	0.7185
15	$SI=1.3+1.866\times GI^{0.6467}$	2.146	40	$SI=1.3+0.6216\times GI^{0.9307}$	0.6799
16	$SI=1.3+1.749\times GI^{0.6623}$	2.060	41	$SI=1.3+0.6003\times GI^{0.9406}$	0.6330
17	$SI=1.3+1.688\times GI^{0.6706}$	1.995	42	$SI=1.3+0.5830\times GI^{0.9490}$	0.6005
18	$SI=1.3+1.584\times GI^{0.6862}$	1.901	43	$SI=1.3+0.5639\times GI^{0.9585}$	0.5460
19	$SI=1.3+1.513\times GI^{0.6976}$	1.807	44	$SI=1.3+0.5475\times GI^{0.9670}$	0.4978
20	$SI=1.3+1.461\times GI^{0.7064}$	1.729	45	$SI=1.3+0.5365\times GI^{0.9732}$	0.4319
21	$SI=1.3+1.425\times GI^{0.7130}$	1.676	46	$SI=1.3+0.5286\times GI^{0.9783}$	0.3500
22	$SI=1.3+1.370\times GI^{0.7230}$	1.636	47	$SI=1.3+0.5291\times GI^{0.9792}$	0.2522
23	$SI=1.3+1.328\times GI^{0.7310}$	1.606	48	$SI=1.3+0.5217\times GI^{0.9843}$	0.1667
24	$SI=1.3+1.279\times GI^{0.7404}$	1.554	49	$SI=1.3+0.5076\times GI^{0.9927}$	0.09031
25	$SI=1.3+1.236\times GI^{0.7491}$	1.520	50	$SI=1.3+0.4952\times GI^{1.000}$	0.02592

TABLE 3 *Parameter estimates[3] and their standard errors for model 3*

Parameter	Estimate	Standard error
b_0	8346000	551400
b_1	3.232	0.02232

3 The parameters were scaled down when fitting the model to ease convergence. The parameter estimates and the standard errors that are reported in this table are re-scaled back to their unscaled magnitude.

fitted model. The parameter estimates for u_i are not shown because they are nuisance parameters and are not used when making height estimates with the model. The value of two of the parameters in the model are unusually large when compared to other models currently used in British Columbia, and may appear to be unreasonable, but they are correct. The mean of the residuals is not significantly different from 0 at $\alpha = 0.05$, and the plots of the residuals did not show any severe heteroscedasticity. The quantile-quantile plot indicated that the residuals may not be normally distributed, but the departure from normality was small. The Durbin-Watson test for serial correlation indicated that there might be some slight negative autocorrelation ($\alpha = 0.0472$). Generally, the assumptions associated with the analysis were adequately met.

DISCUSSION

This research results in a new height–age model and growth intercept models for amabilis fir on the coast of British Columbia. The complete suite of growth intercept models is given in Table 2. The height-breast height age model is:

$$H = 1.3 + (SI - 1.3) \times \left(\frac{(8{,}346{,}000 + R \times 49.5^{2.232}) \times (BHA - 0.5)^{3.232}}{(8{,}346{,}000 + R \times (BHA - 0.5)^{2.232}) \times 299{,}891} \right)^{1/3}$$

where: $R = \varphi + \sqrt{\varphi^2 + 16{,}692{,}000 \times (SI - 1.3)^3 / 299{,}891}$,

$\varphi = (SI - 1.3)^3 / 49.5$,

H = site tree height (m),

SI = site index (m), and

BHA = breast height age (yrs).

The new height-age model and the model developed by Kurucz (1982) were plotted to evaluate the differences in the two models (Figure 1). The two models are nearly identical over the age range of the data. The only significant departure is at breast height ages greater than 150 years and site indices greater than 30 m. This result should not be surprising since the bulk of the data used in developing the two models was the same. However, it does show that the model form and analysis techniques did not, at least in this case, have much of an impact on

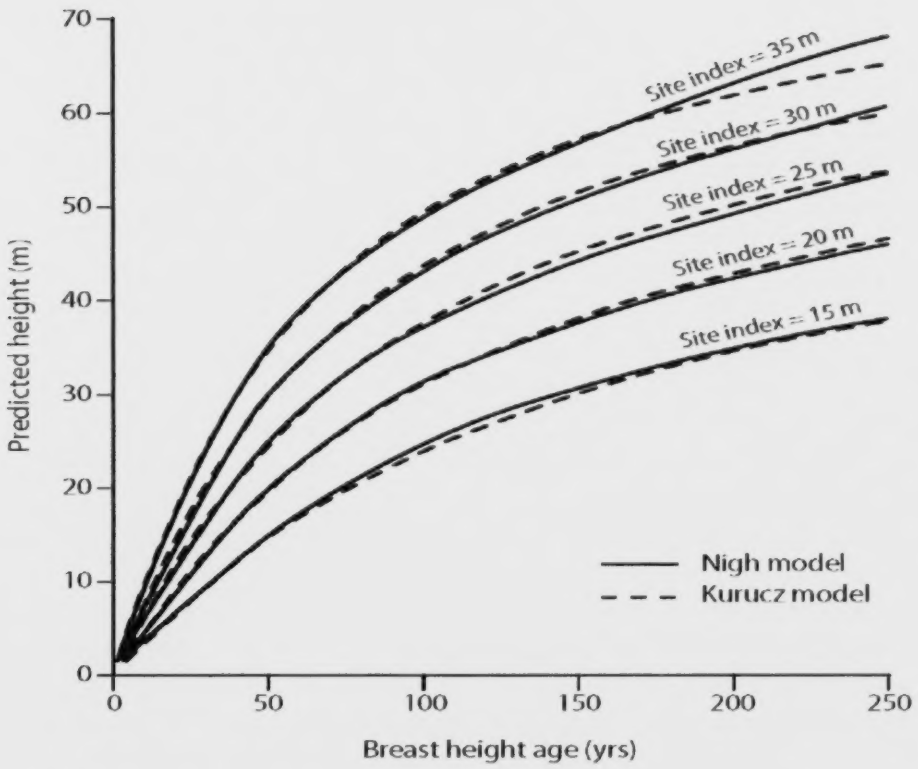

FIGURE 1 *Comparison of the height–age model developed in this project and Kurucz's (1982) height-age model.*

the height predictions.

The similarity between the old and the new height–age models has some positive implications. Implementation of the new height–age models should have minimal consequences on forest management decisions, such as those made in a Timber Supply Review or for silviculture prescriptions and other forest management activities. Also, the fact that the addition of the new data to the data set did not significantly change the curve shape provides evidence that the curves can be safely extrapolated to other geographic areas.

The site index/growth intercept relationship for breast height ages 5, 10, 15, 20, 25, 30, 35, 40, 45, and 50 are shown in Figure 2. This relationship is more curvilinear at young ages, and at breast height age 50, it is a straight line.

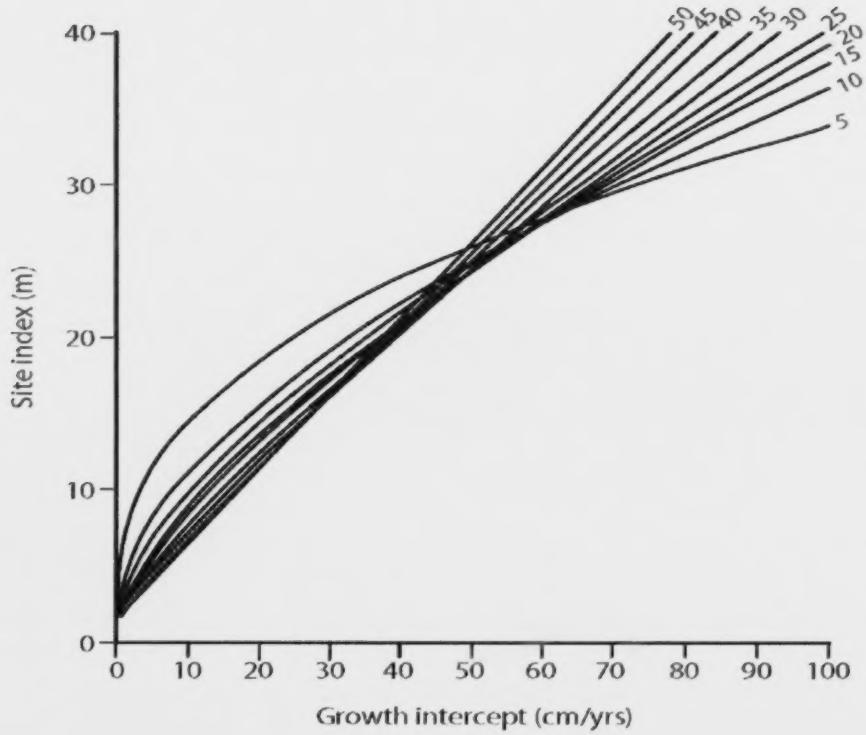

FIGURE 2 *Graphical representation of the growth intercept models for breast height ages 5, 10, 15, 20, 25, 30, 35, 40, 45, and 50.*

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